Review VIII Practical Integral(Slides 555-567)

Integrate!Integrate!Integrate!

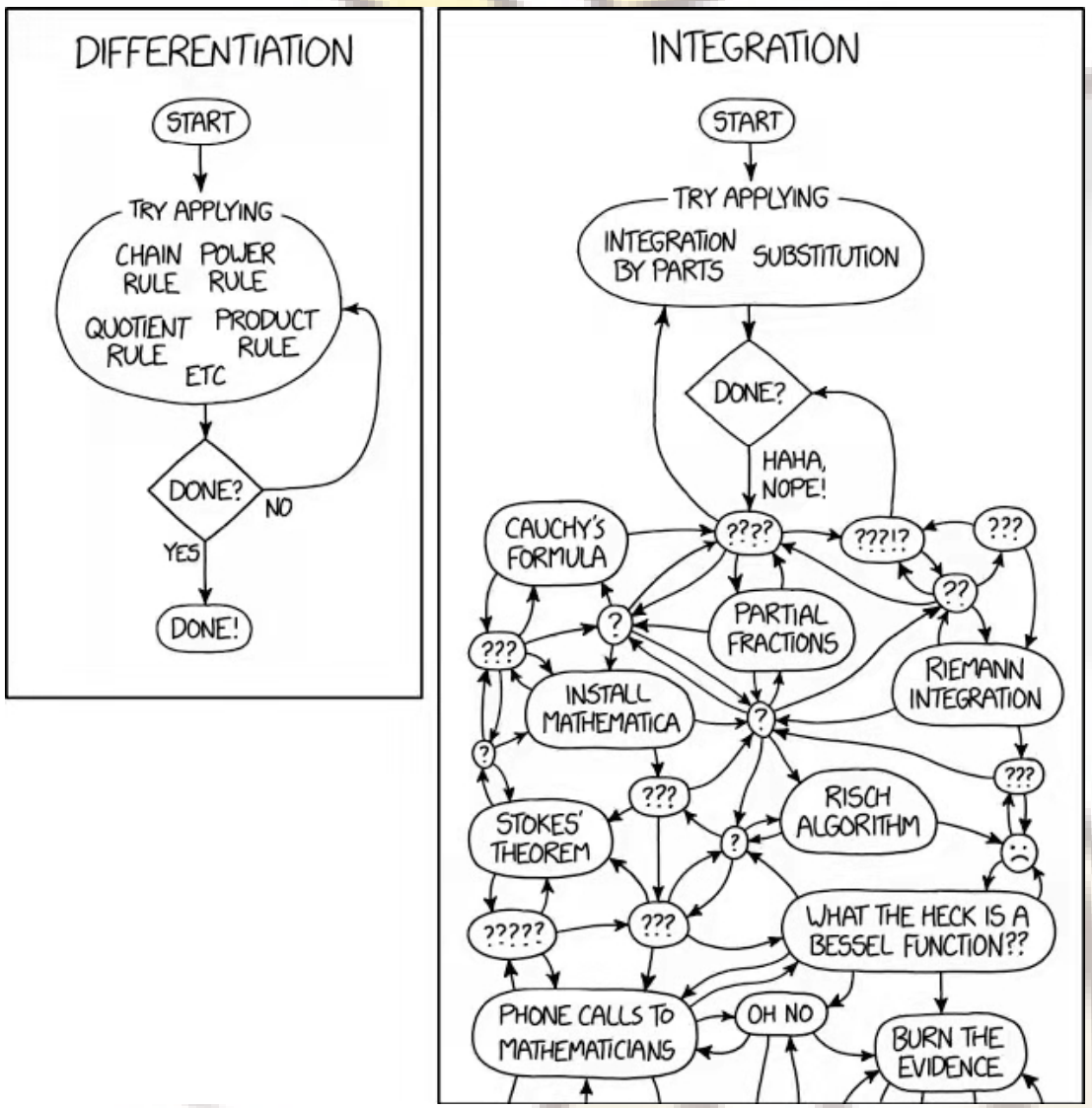
HamHam

University of Michigan-Shanghai Jiao Tong University Joint Institute

December 2, 2021

VV186 - Honors Mathmatics II

Integration



Can you find the original formula?

Integration Method

Method0: Symmetry

Suppose $f(x)$ is an odd integrable function, then

$$\int_{-a}^{a} f(x)dx = 0$$

Exercises: For $a > 0$, calculte

$$\int_{-a}^{a} \frac{\cos(x)}{1 + e(x)^{o(x)}} dx$$

where $e(x)$ is a continuous strictly positive even function, and $o(x)$ is an odd function

Integration Method

Method1: Recite!

Common indefinite integrals include:

- $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C$
- $\int e^{x} dx = e^{x} + C$
- $\int \frac{1}{x} dx = \ln(|x|) + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \ln(x) = ?$

For more complex integrals, we need other theorems to help us evaluate them.

Exercise

Calculte the following integrals:

- $$\int \frac{2}{\sqrt{x}^3} dx$$
- $$\int_3^7 \frac{1}{x^2 + 6x + 5} dx$$

Comment. Partial fraction is sometimes powerful!

Integration Method

Method2: Substitution!

- Let $u = g(x)$.
- Compute $du = g'(x)$.
- Substitute $g(x) = u$ and $g'(x) = du$. **At this moment, only u, no x !!!**
- Calculte the above integral of u, it should be easier.
- Replace u by $g(x)$ to get the result with x.
- For definite integral, pay attention to the range.

Demo

1. $\int sin(x)cos(x)dx$

2. $\int \frac{1}{1+x^2}dx$

Exercise

Calculte the following integrals:

- $$\int_{-1}^{3} \sqrt{9 - x^2}\, dx$$
- $$\int \tan(x)\, dx$$
- $$\int \frac{x}{3x^2 + 6x + 10}\, dx$$
- $$\int \frac{e^{4x}}{1 + e^{2x}}\, dx$$

Integral Method

Method3: Integration by part!

For definite integral:

$$\int_a^b f'(x)g(x)dx = f(x)g(x)\Big|_a^b - \int_a^b f(x)g'(x)dx$$

For indefinite integral:

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

Demo:

$$\int x\sin(x)dx$$

Exercise

- $$\int x^2 e^{-x} dx$$
- $$\int (\ln x)^2 dx$$
- $$\int_0^{\frac{\pi}{2}} \sin^n x \, dx$$

Methodology

1. Substitute what? When to subsitute?

Depend on your luck!

- $\sqrt{a-x^2} \mapsto$ use $x = \sqrt{a}\sin(u)$
- $x^2 + a^2 \mapsto$ use $x = a\tan(u)$
- $x^2 - a^2 \mapsto$ use $x = a\csc(u)$
- Similar terms? Complex terms?
- Other method? Partial fraction?

2. Integrate by part: which to integrate, which to differentiate?

Practice makes perfect!

- Easy and simple terms –> integrate.
- DI table.

Integral Method

Method4: DI Table! (Enhanced version of Method3)

DI Table Method! (Youtube link, VPN required)
Link: `https://www.youtube.com/watch?v=2I-_SV8cwsw`

This is an explanation video on DI method, you can also find other interesting and useful math videos on this channel. After watch the video, calculte

$$\mathcal{L}\sin(x) = \int_0^\infty e^{-px}\sin(bx)dx$$

This is called the *Laplace Transform*.

Some challenge!

More Integration Method? Take VV286!

$$\int \frac{1}{x^4} dx \longmapsto \int \frac{1}{x^4+4} dx \longmapsto \int_{-\infty}^{\infty} \frac{1}{x^4+1} dx$$

Farewell

- Congratulation! You are almost done! Thanks for your hard-working!
- I'm really glad to be your TA, thanks for your support!
- Perhaps I can't be your Vv285 TA... but wish you all the best in your future life and find your own way in JI!

Farewell

- Congratulation! You are almost done! Thanks for your hard-working!
- I'm really glad to be your TA, thanks for your support!
- Perhaps I can't be your Vv285 TA... but wish you all the best in your future life and find your own way in JI!

After VV186: (there are more math classes waiting you)

- Honors Mathematics Sequence: VV186-VV285-VV286
- Linear Algebra: VV214 / VV417
- Discrete Mathematics: VE203
- Probabilistic Methods in Engineering: VE401
- Partial Differential Equations: VV557

another choice is taking vv255!

A simple question¿

Every now and then, ask yourself:

what is "math" based on all the math you have learned in your life?

–question from my TA's TA

Reference

- Exercises from 2020–Vv186 TA-Xia Yuxuan.
- Exercises from Vv286 Assignment and Slides.
- Picture on the door of Dr. Horst Hohberger's Office.
- Mathematical Analysis I. *School of Mathematical Sciences, ECNU*,version 5. Beijing: High Education Press, 2019.5 print.
- Exercise from JI first integration bee.