Review I(Slides 24 - 55)

Logics

"Without logic, mathematics will falls apart... "

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VV186 - Honors Mathmatics II

Statement

First, recall the definition...

- statement
  - true statement
  - false statement
- statement structure
  - quantifier
  - statement frame/predicate
  - specific value
- vacuous truth
  - pink elephants could fly!
  - more examples?

Easy... but be careful!

Definitions? Examples? Notations?

Logical Operation

Type	Logical Operation		Priority
unary	$\neg$	Negation	1
binary	$\wedge$	Conjunction	2
	$\vee$	Disjunction	3
	$\Rightarrow$	Implication	4
	$\Leftrightarrow$	Equivalence	5

- compound statement
  - *tautology*
  - *contradiction*
  - *contingency*

Truth Table

- Can you draw the truth table of *Implication?*
  - How to explain F⇒T?
- How to use a truth table?
  - Understand the problem is about
  - Cover all the possible situations
  - e.g. Prove the following using truth table:(see assignments)

$$\neg(A \wedge B) \Leftrightarrow (\neg A) \vee (\neg B) \qquad (\textbf{\textit{de Morgen rules}})$$

Relations

- Proof by Contradiction

$$(A \Rightarrow B) \equiv \neg(A \wedge \neg B)$$

- Proof by Contraposition

$$(A \Rightarrow B) \equiv (\neg B \Rightarrow \neg A)$$

Quick check:

1. What is **iff**?
2. What is **logically equivalent**?
   A: You take the course Vv186.
   B: I eat breakfast everyday.

$$A \Leftrightarrow B \text{ or } A \equiv B \text{ ?}$$

Logical Quantifiers

Logical Quantifiers		
Sign	Type	Interpretation
$\forall$	universal	for any; for all
$\exists$	existential	there exist; there is some
$\forall \ldots \forall \ldots$	nesting quantifier	for all ...for all ...
$\exists \ldots \exists \ldots$	nesting quantifier	there exists ...(such that) there exist ...
$\forall \ldots \exists \ldots$	nesting quantifier	for any ..., there exists ...
$\exists \ldots \forall \ldots$	nesting quantifier	there exists ...(such that) for any ...
...	...	...

- Try use your own words to interpret them!(see exercises)
- Order matters!

Sets

- What is a set?
- Common Type
  - Empty set: $\emptyset := \{x : x \neq x\}$
  - Total set
  - Subset
  - Proper subset
  - Power set(finite)
- Cardinality(finite)

Quick cheak:

Let $X = \{x : P(x)\}$. Is $P(x)$ a statement?

Operations on Sets

Let

$$A := \{1, 2\} \quad B := \{2, 3\} \quad M := \{1, 2, 3, 4, 5\}$$

Set Operations		
$A \cup B$	Union	$\{1, 2, 3\}$
$A \cap B$	Intersection	$\{2\}$
$A \setminus B$	Difference	$\{1\}$
A^c	Complement	$\{3, 4, 5\}$

- The notation $A - B$ is also used for $A \setminus B$ and $\bar{A}$ for A^c

Ordered Pairs

- What is an ordered pair?
- What is the difference between ordered pair and set?
- Concept of *Cartesian product*.

$$A \times B := \{(a, b) : a \in A, b \in B\}$$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$$

Exercises

1. Let A, B, C be three statements. Use truth table to prove that

$$(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$$

2. Let A, B, C be three sets. Prove that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

- From Exercise 1 & 2, we can see that sets and statements are similar.
- Venn diagram?

Exercises

3. Check whether the following sentences are true statement, false statement, or not a statement.

- $\forall x, y \in \mathbb{R}, x^2 + y^3 \geq 0$
- Let $f(a) = a^4$, then $f(0) > 0$
- For any $a \in \mathbb{R}, a^4 > 0$
- An African Elephant is very big.
- Let A, B be two statements, then $(A \vee B) \Leftrightarrow \neg(\neg A \wedge \neg B)$

Exercises

4. Use quantifiers to rewrite the following definition of covergence:

Let $(a_n)_{n\in\mathbb{N}}$ be a real sequence. If for some fixed $a \in \mathbb{R}$, for any $\varepsilon > 0$, there is an $N \in \mathbb{N}$, such that for all $n > N$, $|a_n - a| < \varepsilon$, then we say (a_n) converges to a.

What's the negation of this statement?

Exercises

5. Interpret the following statement with your own words:

$$\forall x(H(x) \lor \exists y(H(y) \land F(x,y)))$$

Here $H(x)$ means "x takes Vv186", $F(x,y)$ means "x and y are friends", both x and y represent a freshman student in JI.

Exercises

6*. Sheffer stroke & Peirce arrow (also see in assignments)

p NAND q: false iff p&q are both true, marked as $p \mid q$
p NOR q : true iff p& are both false, marked as $p \downarrow q$

Do the following:

- Draw the truth table of NAND and NOR
- Prove that $p \downarrow q$ is logically equivalent to $\neg(p \vee q)$
- Prove taht $p \mid q$ is equivalent to $q \mid p$
- Use the operator $\downarrow$ only to construct the statement $p \Rightarrow q$

Reference

- Exercises from 2019–Vv186 TA-Zhang Leyang.
- Exercises from 2020–Vv186 TA-Xia Yuxuan.
- Exercises from 2020–Vv186 TA-Hu Pingbang.
- Kenneth, H.Rosen. Translated by Xu Liutong etc. *Discrete Mathematics amd Its Applications*, Eighth Edition, Chinese Abridgement. China Machine Press, 2019 print.