Review I(Slides 24 - 55)

Logics

"Without logic, mathematics will falls apart..."

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Statement

First, recall the definition...

- statement
 - ▶ true statement
 - ▶ false statement
- statement structure
 - quantifier
 - statement frame/predicate
 - ► specific value
- vacuous truth
 - pink elephants could fly!
 - ▶ more examples?

Easy... but be careful!

Definitions? Examples? Notations?

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Type	Logical Operation	Priority
unary	¬ Negation	1
binary	∧ Conjunction	2
	∨ Disjunction	3
	\Rightarrow Implication	4
	\Leftrightarrow Equivalence	5

- compound statement
 - ► tautology
 - contradiction
 - contingency

Truth Table

- Can you draw the truth table of *Implication?*
 - ▶ How to explain $F \Rightarrow T$?
- How to use a truth table?
 - ▶ Understand the problem is about
 - Cover all the possible situations
 - e.g. Prove the following using truth table: (see assignments)

$$\neg (A \land B) \Leftrightarrow (\neg A) \lor (\neg B) \quad (de Morgen rules)$$

Relations

• Proof by Contradiction

$$(A \Rightarrow B) \equiv \neg (A \land \neg B)$$

• Proof by Contraposition

$$(A \Rightarrow B) \equiv (\neg B \Rightarrow \neg A)$$

Quick check:

- What is **iff**?
- 2 What is logically equivalent?

A: You take the course Vv186.

B: I eat breakfast everyday.

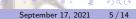
$$A \Leftrightarrow B \text{ or } A \equiv B$$
?

Logical Quantifiers

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Logical Quantifiers				
Sign	Type	Interpretation		
\forall	universal	for any; for all		
3	existential	there exist; there is some		
$\forall \dots \forall \dots$	nesting quantifier	for allfor all		
∃∃	nesting quantifier	there exists(such that) there exist		
$\forall \dots \exists \dots$	nesting quantifier	for any, there exists		
$\exists \dots \forall \dots$	nesting quantifier	there exists(such that) for any		
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- Try use your own words to interpret them! (see exercises)
- Order matters!



Sets

- What is a set?
- Common Type
 - ightharpoonup Empty set: $\emptyset := \{x : x \neq x\}$
 - ► Total set
 - Subset
 - Proper subset
 - ▶ Power set(finite)
- Cardinality(finite)

Quick cheak:

Let $X = \{x : P(x)\}$. Is P(x) a statement?

Operations on Sets

Let

$$A := \{1, 2\}$$
 $B := \{2, 3\}$ $M := \{1, 2, 3, 4, 5\}$

Set Operations			
$A \cup B$	Union	{1, 2, 3}	
$A \cap B$	Intersection	{2}	
$A \setminus B$	Difference	{1}	
A^c	Complement	{3, 4, 5}	

• The notation A - B is also used for $A \setminus B$ and \bar{A} for A^c

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Ordered Pairs

- What is an ordered pair?
- What is the difference between ordered pair and set?
- Concept of Cartesian product.

$$A \times B := \{(a, b) : a \in A, b \in B\}$$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$$

1. Let A, B, C be three statements. Use truth table to prove that

$$(A \lor B) \land C \equiv (A \land C) \lor (B \land C)$$

2. Let A, B, C be three sets. Prove that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

- From Exercise 1 & 2, we can see that sets and statements are similar.
- Venn diagram?

- 3. Check whether the following sentences are true statement, false statement, or not a statement.
 - $\forall x, y \in \mathbb{R}, x^2 + y^3 \ge 0$
 - Let $f(a) = a^4$, then f(0) > 0
 - For any $a \in \mathbb{R}$, $a^4 > 0$
 - An African Elephant is very big.
 - Let A, B be two statements, then $(A \vee B) \Leftrightarrow \neg(\neg A \wedge \neg B)$

4. Use quantifiers to rewrite the following definition of covergence:

Let $(a_n)_{n\in\mathbb{N}}$ be a real sequence. If for some fixed $a\in\mathbb{R}$, for any $\varepsilon>0$, there is an $N\in\mathbb{N}$, such that for all n>N, $|a_n-a|<\varepsilon$, then we say (a_n) converges to a.

What's the negation of this statement?

5. Interpret the following statement with your own words:

$$\forall x (H(x) \vee \exists y (H(y) \wedge F(x,y)))$$

Here H(x) means "x takes Vv186", F(x, y) means "x and y are friends", both x and y represent a freshman student in JI.

6*. Sheffer stroke & Peirce arrow (also see in assignments)

p NAND q: false iff p&q are both true, marked as $p \mid q$ p NOR q: true iff p& are both false, marked as $p \downarrow q$

Do the following:

- Draw the truth table of NAND and NOR
- Prove that $p \downarrow q$ is logically equivalent to $\neg (p \lor q)$
- Prove taht $p \mid q$ is equivalent to $q \mid p$
- Use the operator \downarrow only to construct the statement $p \Rightarrow q$

Reference

- Exercises from 2019–Vv186 TA-Zhang Leyang.
- Exercises from 2020-Vv186 TA-Xia Yuxuan.
- Exercises from 2020-Vv186 TA-Hu Pingbang.
- Kenneth, H.Rosen. Translated by Xu Liutong etc. *Discrete Mathematics amd Its Applications*, Eightth Edition, Chinese Abridgement. China Machine Press, 2019 print.